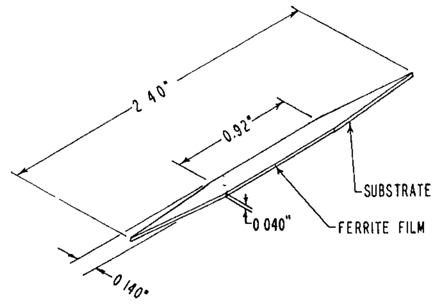

Fig. 1. Ferrite-dielectric configuration.

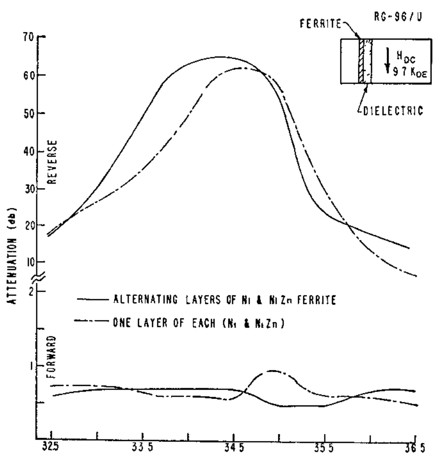

Fig. 2. Isolation characteristics.

alternating layers of Ni and NiZn (74 alternating layers resulting in 41.5 microns ferrite thickness), and 2) the deposition of 20.75 microns of Ni film on a substrate and then an additional deposition of 20.75 microns of NiZn film, resulting in a composite layer of 41.5 microns thickness.

The large reverse-to-forward loss ratios and increased bandwidth obtained with the new multilayer ferrite films represent a significant improvement in nonreciprocal properties over the single-layer ferrite films. The techniques for depositing these two different ferrite films can now be extended to include several different compositions in order to obtain high reverse attenuation over considerable bandwidths with low insertion losses.

It is also to be noted that chemically deposited multilayer ferrite films will have advantages over thin ferrite bulk sections for applications where the magnetic field required is fixed and unsuitable for broadbanding purposes such as in traveling-wave masers.

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References

[1] Wade, W., Jr., T. Collins, and R. Stern, Millimeter resonance isolator using chemically deposited ferrite films, *IRE Trans. on Microwave Theory and Techniques (Correspondence)*, vol MTT-10, Nov 1962, p 611.
[2] Wade, W., Jr., T. Collins, W. W. Malinofsky, and W. Skudera, Chemically deposited thin ferrite films, *J. Appl. Phys.*, vol 34, Apr 1963, pp 1219–1220.

Temperature Calibration of Microwave Thermal Noise Sources

The theory of microwave thermal terminations is discussed with emphasis on equivalent noise temperature calibrations.

The general expression is given for the equivalent noise temperature with an arbitrary temperature and loss distribution. This expression is solved for a constant loss various temperature distributions, and the results are tabulated. An error analysis is presented to show the importance of the insertion-loss calibrations.

An example is given showing a liquid-helium-cooled S-band termination calibrated with these techniques. The input equivalent noise temperature is determined to an accuracy of better than 0.1°K.

Introduction

Thermal noise sources of known absolute equivalent noise temperatures are needed [1]–[3] for radiometry, antenna temperature measurements, and low-noise amplifier performance evaluation. One form of thermal noise source consists of a uniform transmission line with distributed temperature and power loss terminated by a matched resistive element. Nyquist's theorem states that the available thermal noise power from the termination is given by kTB (assuming $hf/kT \gg 1$) where

k = Boltzmann's constant (1.38×10^{-23} J/°K)
T = temperature of the termination in °K
B = bandwidth in c/s
f = operating frequency in c/s
h = Planck's constant (6.624×10^{-34} J/s).

A method is presented for calculating the increase of equivalent noise temperature of a cooled microwave termination due to distributed temperature and transmission-line attenuation. The general equation is solved for various applicable temperature and attenuation distributions, and approximate expressions are given for low-loss transmission lines.

It is assumed that the transmission line is terminated in a matched load so that mismatch errors [4], [5] can be neglected. The temperature calibration error resulting from the transmission-line-attenuation measurement error is shown to be about 0.010°K for each 0.002 dB.

Transmission Line With Distributed Temperature and Attenuation

The differential equation for a traveling wave of power P toward positive x (Fig. 1) in a transmission line with attenuation $\alpha(x)$ and temperature $T_L(x)$ is [6, Eq. (20)].

$$dP/dx = -2\alpha(x)P + 2\alpha(x)kT_L(x)B \quad (1)$$

Substituting $P = kT_xB$ from Nyquist's theorem and dividing by kB, the differential equation in terms of equivalent noise temperature T_x is

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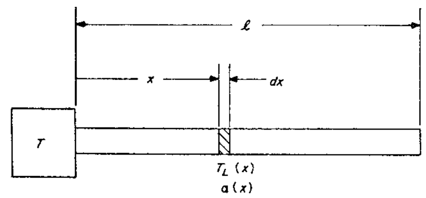

Fig. 1. Lossy transmission line, with attenuation and temperature each a function of position.

$$(dT_x/dx) + 2\alpha(x)T_x = 2\alpha(x)T_L(x) \quad (2)$$

This is a linear first-order differential equation with the solution [7]

$$T' = \frac{T + \int_0^l 2\alpha(x)T_L(x)e^{\int 2\alpha(x)dx}dx}{\left[e^{\int 2\alpha(x)dx}\right]_{x=l}} \quad (3)$$

for a transmission line of length l, with input and output equivalent noise temperatures T and T'.

If $T_L(x) = T_0$ and $\alpha(x) = \alpha$ are constant,

$$e^{\int 2\alpha dx} = e^{2\alpha x}$$

and

$$T' = \frac{T + T_0(e^{2\alpha l} - 1)}{e^{2\alpha l}} = Te^{-2\alpha l} + T_0(1 - e^{-2\alpha l}) \quad (4)$$

or

$$T' = T + (T_0 - T)(1 - e^{-2\alpha l}) = T_0\left(1 - \frac{1}{L}\right) + \frac{1}{L}T$$

Here, $e^{2\alpha l}$ is the insertion-loss ratio L.

If the transmission-line loss is small $2\alpha l \ll 1$,

$$e^{-2\alpha l} = 1 - 2\alpha l + \frac{(2\alpha l)^2}{2} + \cdots$$

and

$$T' = T + (T_0 - T)2\alpha l - \frac{(T_0 - T)}{2}(2\alpha l)^2 + \cdots \quad (5)$$

Since

$$2\alpha l = \frac{L(\text{dB})}{10 \log_{10} e} \simeq \frac{L(\text{dB})}{4.343}$$

we have

$$T' \simeq T + (T_0 - T)\frac{L(\text{dB})}{4.343} - \frac{(T_0 - T)}{2}\left[\frac{L(\text{dB})}{4.343}\right]^2 + \cdots \quad (6)$$

The terms involving L^2(dB) and higher order are normally dropped in calculations, but the L^2(dB) term is retained here for use in evaluating the error involved in the series expansion. For example, with $L = 0.1$ dB, $T_0 = 290$°K, and $T = 0$°K, the error is less than 0.1°K.

For the case with constant attenuation $\alpha(x) = \alpha$, but retaining an arbitrary temperature distribution, (3) gives

$$T' = \frac{T + \int_0^l 2\alpha e^{2\alpha x}T_L(x)dx}{e^{2\alpha l}} \quad (7)$$

With constant attenuation and a linear tem-

TABLE I

THEORETICAL EQUIVALENT NOISE TEMPERATURES OF A TERMINATION ATTENUATED BY TRANSMISSION LINES WITH VARIOUS COMBINATIONS OF THERMAL TEMPERATURE DISTRIBUTIONS

Transmission-line conditions	Unknown	Exact	Approximation (valid if transmission-line loss is less than approximately 0.5 dB)
Transmission line with loss L and constant temperature distribution T_0	T'	$T + (T_0 - T)\left(\ - \frac{1}{L}\right)$	$T + (T_0 - T)\mathcal{L} - \frac{1}{2}(T_0 - T)\mathcal{L}^2 + \cdots$
	$(T' - T_0)$	$(T - T_0)\left(\frac{1}{L}\right)$	$(T - T_0)(1 - \mathcal{L} + \frac{1}{2}\mathcal{L}^2 + \cdots)$
Transmission line with loss L and linear temperature distribution from T to T_0	T'	$T + (T_0 - T)\left(1 - \frac{1 - \frac{1}{L}}{\mathcal{L}}\right)$	$T + \frac{(T_0 - T)}{2}\mathcal{L} - \frac{(T_0 - T)}{6}\mathcal{L}^2 + \cdots$
	$(T' - T_0)$	$(T - T_0)\left\{\frac{1 - \frac{1}{L}}{\mathcal{L}}\right\}$	$(T - T_0)(1 - \frac{1}{2}\mathcal{L} + \frac{1}{6}\mathcal{L}^2 + \cdots)$
Transmission line with loss L_1 and linear temperature distribution from T to T_0 for Sec. 1, and loss L_2 and constant temperature distribution T_0 for Sec. 2	T'	$T + (T_0 - T)\left[1 - \frac{\left(1 - \frac{1}{L_1}\right)\left(\frac{1}{L_2}\right)}{\mathcal{L}}\right]$	$T + (T_0 - T)(\frac{1}{2}\mathcal{L}_1 + \mathcal{L}_2)$ $- (T_0 - T)(\frac{1}{6}\mathcal{L}_1^2 + \frac{1}{2}\mathcal{L}_2^2 + \frac{1}{2}\mathcal{L}_1\mathcal{L}_2) + \cdots$
	$(T' - T_0)$	$(T - T_0)\left[\frac{\left(1 - \frac{1}{L_1}\right)\left(\frac{1}{L_2}\right)}{\mathcal{L}_1}\right]$	$(T - T_0)[1 - (\frac{1}{2}\mathcal{L}_1 + \mathcal{L}_2)$ $+ (\frac{1}{6}\mathcal{L}_1^2 + \frac{1}{2}\mathcal{L}_2^2 + \frac{1}{2}\mathcal{L}_1\mathcal{L}_2) + \cdots]$

$$T' = \frac{T + \frac{\mathcal{L}}{l}\int_0^l \exp\left(\frac{\mathcal{L}x}{l}\right) T_L(x)dx}{L}$$

for a transmission line with loss L independent of temperature, length l, and temperature distribution $T_L(x)$, where

$$\mathcal{L} = \frac{L\ (\text{dB})}{10 \log_{10} e} \simeq 0.23026L\ (\text{dB})$$

T, T' = input and output equivalent noise temperatures

perature distribution along the transmission line $T_L = T + (T_0 - T)x/l$, (7) gives

$$T' = \frac{T + \int_0^l 2\alpha e^{2\alpha x}\left[T + (T_0 - T)\frac{x}{l}\right]dx}{e^{2\alpha l}} \qquad (8)$$

Integrating (8),

$$T' = T + \frac{(T_0 - T)}{2\alpha l}(2\alpha l - 1 + e^{-2\alpha l}) \qquad (9)$$

If the transmission-line loss is small, $2\alpha l \ll 1$,

$$e^{-2\alpha l} = 1 - 2\alpha L + \frac{(2\alpha l)^2}{2} - \frac{(2\alpha l)^3}{6} + \cdots$$

and

$$T' \simeq T + \frac{(T_0 - T)}{2}\frac{L(\text{dB})}{4.343} - \frac{(T_0 - T)}{6}\left[\frac{L(\text{dB})}{4.343}\right]^2 + \cdots \qquad (10)$$

Comparing (10) with (6), note the appearance of average temperature, $(T_0 - T)/2$. Again the terms involving L^2(dB) and higher are usually dropped in calculations.

A common simplification made in the calibration of equivalent noise temperatures of microwave terminations is to consider the termination in a reference bath of temperature T (hot or cold) separated by a transmission line of loss L_1 with a linear thermal temperature distribution between T and T_0. This is separated by another transmission line of loss L_2 at a thermal temperature T_0. If the transmission-line losses are low, the calibrated temperature from (6) and (10) accounting for both transmission lines and retaining up to second-order terms is

$$T' = T + (T_0 - T)\left[\frac{L_1(\text{dB})}{8.686} + \frac{L_2(\text{dB})}{4.343}\right] - (T_0 - T)\left\{\frac{1}{6}\left[\frac{L_1(\text{dB})}{4.343}\right]^2 + \frac{1}{2}\left[\frac{L_2(\text{dB})}{4.343}\right]^2 + \frac{1}{2}\left[\frac{L_1(\text{dB})}{4.343}\right]\cdot\left[\frac{L_2(\text{dB})}{4.343}\right]\right\} + \cdots \qquad (11)$$

Exact and approximate solutions to (7) are tabulated in Table I for various typical transmission-line temperature distributions. The approximations are especially useful where the transmission-line loss is known in dB, and are quite accurate when the losses are low.

CALIBRATION MEASUREMENT ERROR

The most critical measurement in the calibration of the equivalent noise temperature of a reference termination is the insertion loss of the transmission line. For example, if the temperature distribution is constant (T_0) along a transmission line with loss L, the equivalent noise temperature (Table I) is

$$T' \simeq T + 0.2303(T_0 - T)L(\text{dB}) \qquad (12)$$

The error in T' due to insertion-loss measurement errors is [differentiating (12)]

$$\Delta T \simeq 0.2303(T_0 - T)\Delta L(\text{dB}) \qquad (13)$$

Equation (13) is plotted in Fig. 2 for a liquid-helium and a liquid-nitrogen-cooled termination. To determine T' to an accuracy of 0.1°K for a liquid-nitrogen-cooled termin-

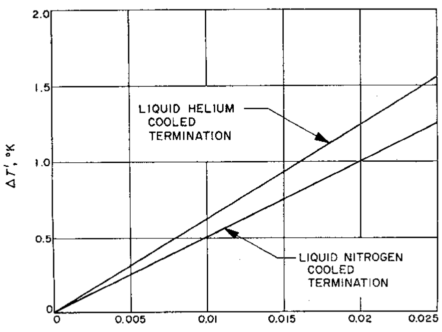

Fig. 2. Calibration error in a reference termination due to insertion-loss measurement error.

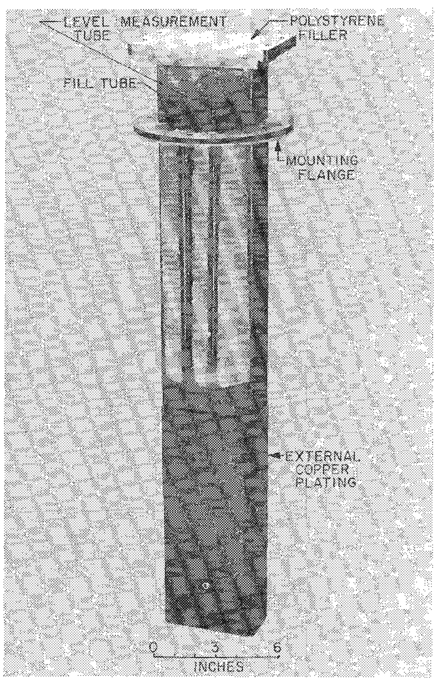

Fig. 3. S-band WR-430 waveguide liquid-helium-cooled termination.

ation requires better than 0.002-dB accuracy in the insertion-loss measurements [8].

Experimental Calibrated Thermal Termination

An S-band (Fig. 3) waveguide liquid-helium-cooled thermal termination [3] was calibrated to high precision using (11). Waveguide construction was used to minimize losses. This termination is normally installed in a 10-liter-glass Dewar, and has about 10 hours of operating life. The stainless-steel section of copper-plated waveguide located between the external copper-plated section containing the cooled termination and the mounting flange had an insertion loss of L_1 of 0.009 dB. The outer section of waveguide above the mounting flange had at ambient temperature an insertion loss L_2 of 0.008 dB. (This includes the polystyrene waveguide window.) With reference to the input flange, the equivalent noise temperature is 5.0°K [from (11)] over the frequency ranges where the termination is matched. An accuracy of 10 per cent in the calibration of the transmission line would result in less than 0.1°K error in the equivalent noise temperature.

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References

[1] Schuster, D., C. T. Stelzried, and G. S. Levy, The determination of noise temperatures of large paraboloidal antennas, *IRE Trans. on Antennas and Propagation*, vol AP-10, May 1962, pp 286–291.
[2] Stelzried, C. T., A liquid-helium-cooled coaxial termination *Proc. IRE*, (*Correspondence*), vol 49, Jul 1961, p. 1224.
[3] Clauss, R. C., W. Higa, C. Stelzried, and E. Wiebe, Total system noise temperature: 15°K, *IEEE Trans. on Microwave Theory and Techniques*, (*Correspondence*), vol MTT-12, Nov 1964, pp 619–620.
[4] Beatty, R. W., Mismatch errors in the measurement of ultra high frequency and microwave variable attenuations, *J. Res. Nat. Bur. Std.*, vol 52, no. 1, Res. Paper 2465, 1954.
[5] Daywitt, W. C., Microwave noise, in *National Bureau of Standards Course on Electromagnetic Measurements and Standards*. Boulder, Colo.: U. S. Dept. of Commerce, Jul 1963.
[6] Siegman, A. E., Thermal noise in microwave systems, *Microwave J.*, Mar 1961.
[7] Ford, L. R., *Differential Equations*, 2nd ed., New York: McGraw-Hill, 1955.
[8] Stelzried, C. T., and S. M. Petty, Microwave insertion loss test set, *IEEE Trans. on Microwave Theory and Techniques*, (*Correspondence*), vol MTT-12, Jul 1964, pp 475–477.

Graphical Procedures for Finding Matrix Elements for a Lattice Network and a Section of Transmission Line

Four-terminal networks and two-port junctions can be represented by several different well-known 2 by 2 matrices. Tables relating the elements in the various matrices have been published.[1–5] Graphical procedures are presented here which may be used to determine the elements in these matrices for a lattice network and a section of transmission line, as shown in Fig. 1.

In the discussion which follows,

$$\mathbf{\Gamma}(\mathbf{Z}) = (\mathbf{Z} - 1)/(\mathbf{Z} + 1)$$

and

$$\mathbf{Z}(\mathbf{\Gamma}) = (1 + \mathbf{\Gamma})/(1 - \mathbf{\Gamma})$$

Derivations are omitted because they are simply boring algebraic manipulations.

The elements of the scattering matrix for a lattice network are given by

$$2S_{11} = 2S_{22} = \mathbf{\Gamma}(\mathbf{Z}_a') + \mathbf{\Gamma}(\mathbf{Z}_b')$$
$$2S_{12} = 2S_{21} = -\mathbf{\Gamma}(\mathbf{Z}_a') + \mathbf{\Gamma}(\mathbf{Z}_b')$$

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[1] Guillemin, E. A., *Communication Networks*, vol II. New York: John Wiley, 1935, pp 137–138.

[2] Bolinder, E. F., Note on the matrix representation of linear two-port networks, *IRE Trans. on Circuit Theory*, vol CT-4, Dec 1957, pp 337–339.

[3] Beatty, R. W., and D. M. Kerns, Relationships between different kinds of network parameters, not assuming reciprocity or equality of the waveguide or transmission line characteristic impedances, *Proc. IEEE*, vol 52, Jan 1964, p 84.

[4] Mathis, H. F., Matrix conversion table, *Microwaves*, vol 3, Feb 1964, pp 28–33.

[5] Kopp, E. H., Matrices for basic two-port networks, *Electro-Technology*, vol 73, Mar 1964, pp 34–39.

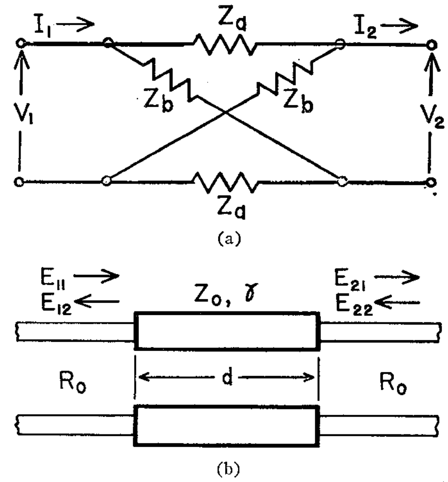

Fig. 1. The two networks considered.

where it is assumed that two transmission lines with characteristic impedance R_0 are connected to the network, $\mathbf{Z}_a' = \mathbf{Z}_a/R_0$, and $\mathbf{Z}_b' = \mathbf{Z}_b/R_0$. A graphic method for finding these elements is illustrated in Fig. 2, and either a Smith or a Carter chart[6,7] may be used.

Logarithmic transmission-line charts[8–10] are used to find the matrix elements for a section of transmission line and a lattice network. The following points are located on the chart, as shown in Fig. 3: E at the left $\mathbf{Z} = \infty$, F at $Z = 0$, G at $\mathbf{Z} = 1 \angle -90° = -j$, and H at the right $\mathbf{Z} = \infty$. The points J and K are plotted at $\mathbf{Z}_J = R_0/\mathbf{Z}_0$, and $\mathbf{Z}_K = \mathbf{Z}_0/R_0 = 1/\mathbf{Z}_J$. The point L is located so that $(\overline{EL})/\overline{EH}) = \alpha/2\beta$, where $\gamma = \alpha + j\beta$. The point M is located on the line LH at the horizontal distance $d/\lambda_g = \beta d/2\pi$ from H as shown. The point N is the midpoint of the line MH. For a lattice network,

$$\mathbf{Z}_K = \sqrt{\mathbf{Z}_a'\mathbf{Z}_b'} = \sqrt{\mathbf{Z}_a\mathbf{Z}_b}/R_0$$
$$\mathbf{Z}_N = \sqrt{\mathbf{Z}_b'/\mathbf{Z}_a'} = \sqrt{\mathbf{Z}_b/\mathbf{Z}_a}$$

and

$$\mathbf{Z}_0 = \sqrt{\mathbf{Z}_a\mathbf{Z}_b}$$

The point P is located on the line HJ so that $\overline{JP} = \overline{HJ}$. The point Q is located on the line FK so that $\overline{KQ} = \overline{FK}$. The lines RG, KS, UJ, QW, and XP are drawn parallel to the line LH. The various points are located so that

$$\overline{RG} = \overline{MN} = \overline{NH} = \overline{KS} = \overline{UV} = \overline{VJ} = \overline{QW} = \overline{XP}$$

and

$$\overline{MU} = \overline{FK} = \overline{KQ} = \overline{NV} = \overline{VX} = \overline{SW} = \overline{HJ} = \overline{JP}$$

[6] Smith, P. H., Transmission-line calculator, *Electronics*, vol 12, Jan 1939, pp 29–31; An improved transmission-line calculator, vol 17, Jan 1944, pp 130–133, 318, 320, 322, 324, 325.

[7] Carter, P. S., Charts for transmission line measurements and computations, *RCA Rev*, vol 3, Jan 1939, pp 355–368.

[8] Cafferata, H., The calculation of input impedance, or sending-end impedance of feeders and cables terminated by complex loads, *Marconi Rev.*, no. 64, Jan–Feb 1937, pp 12–19; no. 67, Sep–Dec 1937, pp 21–39.

[9] Guillien, R., Nouveau diagramme permettant par translation les transformations d'impedances, *L'Onde Electrique*, vol 35, Dec 1955, pp 1164–1170.

[10] Mathis, H. F., Logarithmic transmission-line charts, *Electronics*, vol 34, Dec 1, 1961, pp 48, 50, 52, 54.